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CPUE Analysis CJM purse seine fishery center-south Chile 1994-2020

Chile

CPUE Analysis of the jack mackerel purse-seine fishery of the center-south zone of Chile, June 1994 through July 2020

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Summary

Chile provides a relevant set of data for jack mackerel stock assessment to be conducted by the SPRFMO. Out of this data, the standardization of fishing operations of the Chilean fleet using catch data at a level of fishing trips. Since fishing trips may hide resolution on fishing operations and its strategies, an integration of information was conducted to develop a catch per set database for the 1994-2020 period. This database was used to apply a standardized statistic model that includes a distribution of compound probability that describes the joint probability of success and a catch per set fishing. The estimated average values of catch per set are subject to interpretation in terms of fishing milestones and the annual predictors allow the traceability of the CPUE trend. This new model predicts similar results to the standardized CPUE that Chile annually contributes to the SPRFMO. However, some precautions shall be noted on the differences detected.

INTRODUCTION

The standardized jack mackerel catch per unit of effort (CPUE) has covered the situation overtime because of situations that have a great impact I the management of this fishery. Therefore, seasonal variations of the CPUE described through a standardization model, expressed as a relative index of abundance allows helping the calibration of the stock assessment model and after that, to the management regulation have social and economic relevance. This is especially significant in a fishery that has the participation of multiple players (Members) and it is managed at a regional level by the South Pacific Regional Fisheries management Organization (SPRFMO).

Two elements to describe seasonal variations of the CPUE shall be taken into account. The first is the selection of a model that allows the proper description of its seasonal variations and the latter, on the other hand, is the quality of the set of data used to describe the CPUE average values. It is crucial to have an adequate model and a database as complete as possible.

Chile, as a SPRFMO member, has contributed with a standardized CPUE time series using appropriate models to describe the CPUE variations. This series has included the fishing trips as a sampling unit, quantifying the catch and average effort according to the days out of port and the displaced holding capacity (Saavedra-Nievas *et al*. 2012). In this work and unlike the previous versions, the fishing operation and the jack mackerel catch in associated tones is considered as a sampling and effort unit, assuming that the gears night contain a better representation on the changes of the CPUE. This study is included in a cooperation and collaborative work between the

Fisheries Development Institute (IFOP) and the Fisheries Research Institute (INPESCA) in coordination with the Jack Mackerel Technical-Scientific Committee (CCT-J).

BACKGROUND

Location of the jack mackerel fishery

Center-south Purse-seine pelagic fishery takes into account the maritime space including the north-south limits between 32°10' and 47°00' SL, while the area from the East to West includes the Exclusive Economic Zone (EEZ) of Continental Chile up to 1,200 nm in the high seas, including Ocean Islands. The north-south area is divided in five fishing grounds (San Antonio, Talcahuano, Valdivia, Chiloé, and Guaitecas). The Guaitecas area was expanded in a degree in a southerly direction in 2008, due to some fishing incursions of the purse seine industrial fleet towards higher latitudes (Aysen Region) conducted in the autumn amongst 2008-2011 and in May 2015. This analysis also includes the areas of Caldera and Coquimbo, located North of parallel 32°S, since the center-south fleet conducted sets in such areas for some years, although in occasional times. The geographical demarcation of fishing grounds and maritime areas that allow the conduction of operational analysis at a spatial-temporal level of the fishing activity is shown in **Figure 1** and **Table 1.**

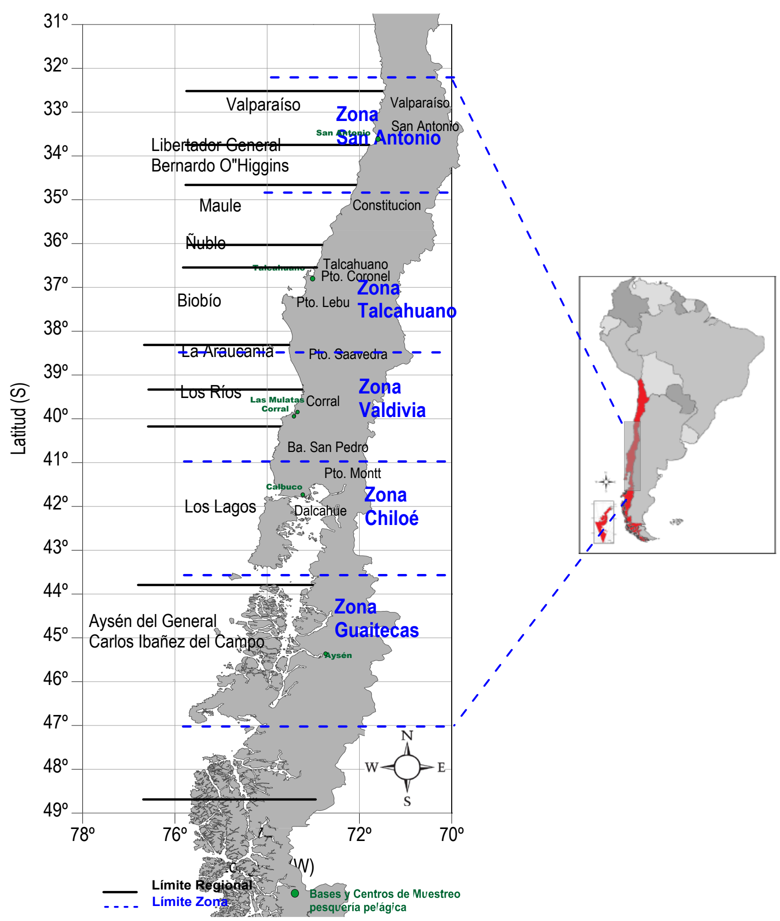

Figure 1. Monitoring area of the pelagic purse seine fishery from the center-south macro region with the subdivision of fishing grounds.

Table 1

Fishing grounds established for the Center-South macro region

Fishing grounds	latitude (°S)
Caldera	26°00’ – 28°00’
Coquimbo	28°00’ – 32°10’
San Antonio	32°10’ – 34°50’
Talcahuano	34°50’ – 38°30’
Valdivia	38°30’ – 41°00’
Chiloé	41°00’ – 43°30’
Guaitecas	43°30’ – 47°00’

3. Evolution and characteristics of the industrial purse-seine fleet

The purse-seine industrial fleet in the Center-South macro area has shown significant changes over time both regarding its capture composition, size, structure, and technology. These modifications place it amongst the most modern and efficient of the world, sharing the podium with Norway, Island and Denmark (**Figure 2**). The fleet has been over different stages in accordance with the evolution of the fishery and its main resource, that is, jack mackerel. First, it showed a strong and sustained growth, in number and capacity of fishing vessels and then a dramatic fall in size, adjusting to the lower abundance of the resource and the smaller fishing quotas. Therefore, companies focused in merge processes and structural reorganization with new operational strategies aimed at reducing costs and maximizing economic efficiency.

Purse-seine industrial fleet from the center-south macroarea moved up as the first place at a national level from 1991 in terms of storage capacity to fleet, reaching 131 thousand in terms of hauling capacity, reaching in 1997 131 thousand m^3 (189 vessels), displacing the north macroarea; this situation has persisted for several years until today. The center-southern fleet was larger than the northern fleet with respect to the number of vessels (only between 1994 and 2001) in accordance with the strong growth of the jack mackerel fishery. It showed then a strong and persistent decrease in the number of vessels but with a sustained growth of the average size. This situation is observed thus far (**Figure 3**).

Figure 2. Types of industrial vessels that have operated in the purse-seine fishery of the center-south macroarea, in accordance with the more representative size, between the Valparaíso and Aysen regions, 2001-2019.

Up to June 2020, the purse seine industrial fleet included 26 vessels. They represented a hold capacity accumulated a total of 38.2 thousand m^3 and an average hold capacity of 1,468 m^3. They all concentrated in the Biobío region. At the San Antonio port (Valparaiso), there were high seas fishing owners with their headquarters located there between 1985 and 20000. However, the latter (most of them) were transferred to ports in the Biobio region and after that to *parking boats* that still exist at the cities of Valdivia and Chiloe from 2001.

At Tomé and Talcahuano there is no fleet with headquarters there since 2005 and 2008, respectively. Currently, the size of the center-south industrial purse-seine fleet (26 vessels and 38,1thousand m3) is significantly smaller than the size registered in 2006 (65 vessels and 70,3 thousand m3) as a result of the adjustment after LMCA as of 2001 and the strong reduction of the fishing quotas of the last period with a trend to the growth due to the recovery of the fishery (**Figure 3**).

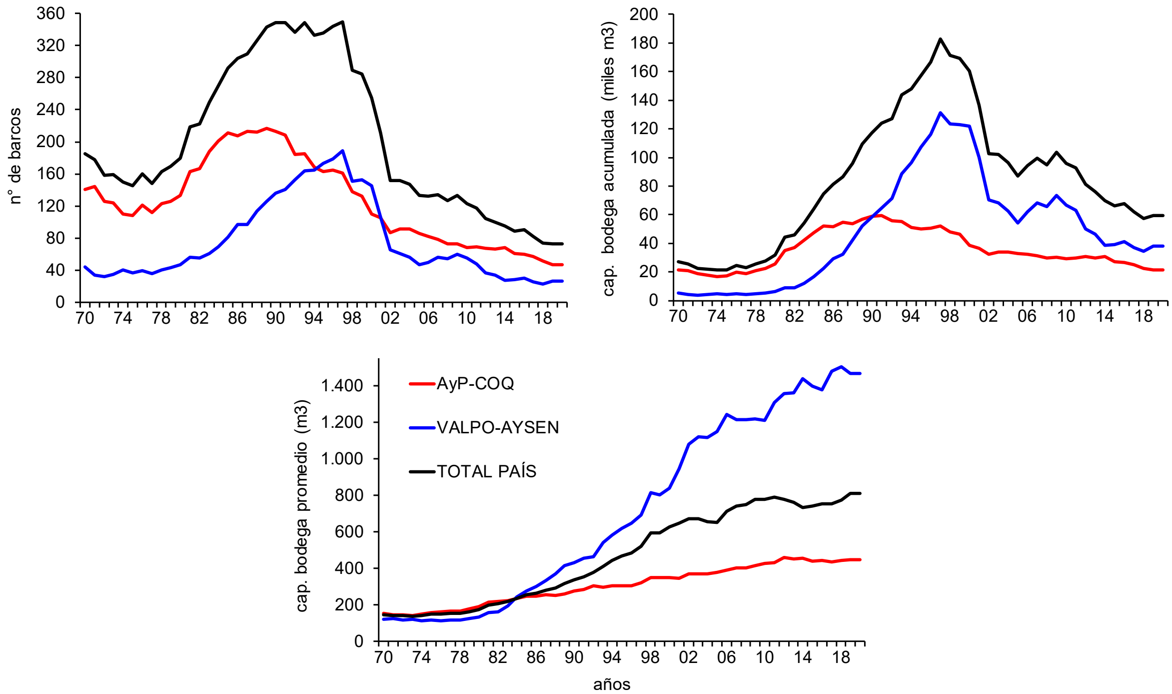

Figure 3. Evolution of the industrial purse seine fishery in the center-south macroarea.

METHODOLOGY

Background of the Information

Jack mackerel catch per set information came from the fishing logbooks collected by the Fisheries Research Institute (INPESCA) and provided by the Fisheries Development Institute (IFOP) for analysis through a cooperation agreement of scientific research. INPESCA, located in the Biobío Region, has implemented a regular and systematized monitoring program for the jack mackerel fishery, with scientific observers onboard that record set by set and, in some fishing trips, chosen randomly, the vessel operational activity. Such information was complemented and integrated in a single data base with fishing logbooks collected by IFOP for the same area and vessels. On the other hand, information needed for the identification of vessels and their technological characteristics was obtained from the database from the National Service for Fisheries and Aquaculture. This information was collected over the years and allows following the modifications in the fleet structure and composition, in terms of the number of operational vessels and their location by port or region. It also serves to determine variations of the main annual dimensions which allow the conduction of short and long term projections.

Data

Daily logbooks of fishing sets associated to the industrial purse seine fleet that operated in the Chilean center-south macroarea in the period that included June 1994 and July 2020 was analyzed. With regards to the spatial changes observed in catches and the increase of the operational areas of the fleet over time, the study area was subdivided into 7 fishing subareas that are located between 20°00´S and 47°00´S Latitude and 70°5'W to 99°20'W Longitude (**Table 1**, **Figure 4**). Intra-annual seasonality of the fishery was assessed through the month-operation factor, while the historic operational dynamics was assessed through the incorporation of two covariates (storage capacity in m^3 and the days out of port of the fishing trip). For analytical purposes, it is assumed that in addition of the operational efficiency of the fleet, the CPUE would represent a measurement of the relative abundance or availability (exploited fraction of the resource stock within the study area)

Statistical Model

The statistical nature of the variable of interest determines the probability model that will serve as a basis for the estimation. Several authors have described catch as a non-negative variable that contains an excessive amount of zeros, especially at the species level, which conditions the use of the model methods based on the assumption of a normal distribution (Ortiz *et al.*, 2000; Dietrich, 2003; Delord *et al.*, 2005).

When catch is recorded in weight, models based on the Δ-Lognormal distribution are commonly used (Aitchison & Brown, 1957; Pennington, 1983) and the Tweedie distribution (Smyth & Jϕrgensen, 2002; Dunn and Smyth, 2005; Zhang, 2013), especially a type of distribution called compound Poisson or compound Poisson-Gamma (Tascheri *et. al.* 2010). Distribution Δ-Lognormal allows to model the behavior of the mean catch by set with capture and the proportion of sets with capture separately. In contrast, the compound Poisson-Gamma distribution allows to simultaneously model the number of sets with capture and the catch rates (sets with positive catch), estimating mean catch by set, which involves an advantage with respect to the separate model. This is the reason for which this type of model was used.

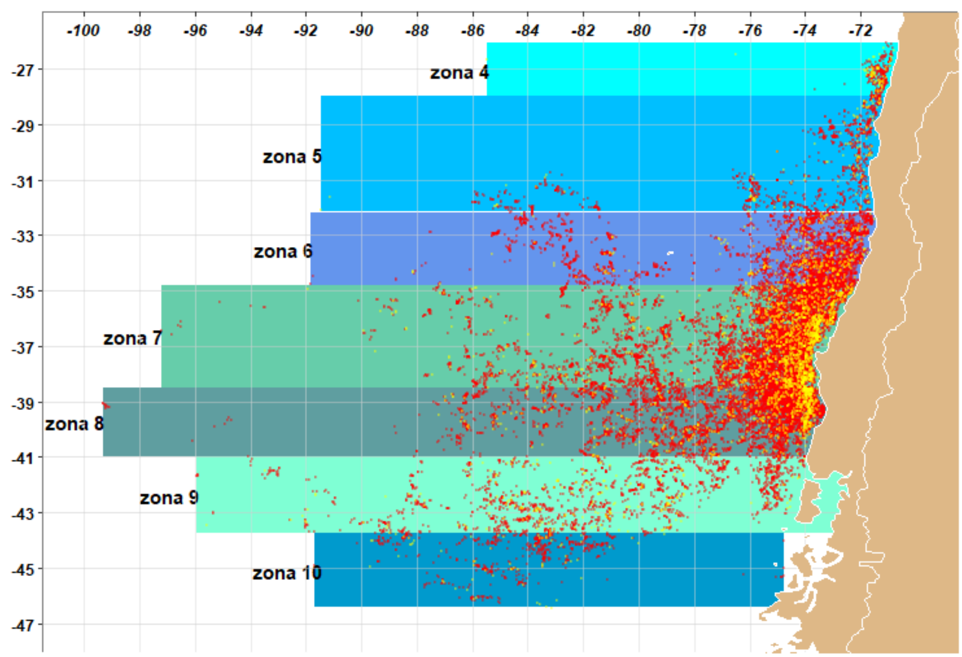

Figure 4. Spatial stratification of the center-south area in fishing subareas and identification of sets with jack mackerel sets with catch (red) and without fishing (yellow) within the period corresponding to June 1994 through July 2020.

Expansion from the sample to the whole effort implies that a unit of effort has the same effect on the catch that any other unit of effort. However, spatial and temporary factors, among others, may have an impact on the result of a fishing set with regards to the catch obtained. With the aim at assessing the effect on these factors in the catch, generalized lineal models were used (GLM, McCullagh & Nelder, 1989) under a Poisson-Gamma distribution composed by (Smyth & Jϕrgensen, 2002. For more details of the formulation where the rate of catch per set is estimated through the different levels of predictor variables (factors and covariates, revise Annex). They are developed by the year, month, and area where the set was conducted, as well as, the storage capacity of the vessel and the days out of port. Therefore, the systematic part of the model has the following form:

$$g(\mu_i) = \beta_0 + \sum_{k=1}^{45} \beta_k x_{ik} \qquad \forall\ i = 1, \dots, m$$

where μ_i represents the mean catch in the ith lance, β_k corresponds to the parameter that measures the linear relation of covariate x_{ik} observed to the ith set through the linking function $g(\cdot)$, the parameter β_0 represents the intercept term in the intercept in the linear equation and m corresponds to the number of total sets on study, where the covariables are:

- x_{i1}: Is the storage capacity (m^3) of the vessel that conducted the i-esh set.
- x_{ik}: Is the indicate variable related to the area where study the set was conducted, where $x_{ik} = 1$ if the i-esh set was conducted in the area k and $x_{ik} = 0$ otherwise, for $k = 2, \dots, 7$.
- x_{ik}: Is a indicate variable related to the year in which the set was conducted, where $x_{ik} = 1$ if the i-esh set was conducted in the corresponding year and $x_{ik} = 0$ otherwise, for $k = 8, \dots, 33$.
- x_{ik}: Corresponds to the indicative variable related to the month in which the set was conducted, where $x_{ik} = 1$ if the i-esh set was conducted in the corresponding month and $x_{ik} = 0$ otherwise for $k = 34, \dots, 44$.
- x_{i45}: Are the days out of port (nº of days in decimal fraction) of the vessel where the i-*esh* set was conducted.

Parameters of the model were estimated through the maximum likelihood model, subject to a fixed value of the power parameter of the variance function (methodological details in Anex), the significance of each of the factors was assessed through the analysis of Devianza. For the processes the software R (R Core Team, 2020) was used; it was also necessary to resort to the statistical packages *tweedie* (Dunn, 2017) and *statmod* (Giner and Smyth, 2016).

Results

Monitoring of the jackmackerel catch of the purse seine industrial fleet in the center-south macrozone during the period June 1994 and July 2020 (INPESCA) registered a total of 95 fishing vessels. They represent 41% of the fleet monitored by IFOP, with a minimum of 8 vessels at the beginning of the series, a maximum of 36 vessels between 1999 and 2009 and an average of 24 (SE=1,6) vessels for the period under study. Such monitored fleet captured near 5 million tons of jack mackerel in that period and showed a high dynamic in its composition and structure through the years, with a sustained growth in the average size A FLOTE of approximately 800 m^3 at the beginning of the period up to 1600 m^3 at the end of the series.

During the assessment period, a total near 8 thousand fishing trips with or without catch of jack mackerel was registered. This number had an annual variation between 32 and 821 fishing trips per year and a mean around 290 trips (SE=40,6). The higher number concentrated in 2002 and between 2004 and 2005 trips were near 800 per year. Regarding the effort exerted in terms of sets, near 30.50 (M=1 129; SE=140) were observed. This number coincided with those years in which trips were more concentrated. While the percentage of sets with catch varied between 60%

and 97%, in 1994 and 2015, respectively. It is important to mention that from 2001, the percentage of sets with catch has maintained at around 90 %, with the exception of 2009, 2011, and 2018 that percentage was almost 85 % of sets with catch. Lastly, the annual average of days out of port varied from 2 to 8.6 days (M=3,8, SE=0,36), with an important increase between 2008 and 2011 (M=7,5; SE=0,44) and to a lesser extent, in 2014 and 2015 (M=5,5; SE=0,26) in contrast to the rest of the series (M=2,9; SE=0,14).

CPUE Model

The power parameter (see Annex for methodological description) of the variance function (p) was estimated in 1,484, which indicates that the underlying distribution to the catch per set rates tends to maintain a compound Poisson-Gamma distribution. The fitted model allows the explication the 10.1% of the total Deviance of the index. Estimation of the model parameters, its standard error, coefficient of variation (CV%) percentage, and individual significance are shown in **Table 2.**

Table 2. Estimated values, its standard error, coefficient of variation, and individual significance of the parameters related to the compound Poisson-Gamma model.

Parameters	Estimation	Standard error	CV(%)	p-value
Intercept	4,630	0,1210	2,60%	< 2e-16
Storage Cap.	0,000	0,0000	11,00%	< 2e-16
Zone 5	0,246	0,0641	26,10%	< 1e-04
Zone 6	0,084	0,0593	70,30%	0,155
Zone 7	-0,035	0,0587	169,00%	0,554
Zone 8	0,024	0,0587	241,80%	0,679
Zone 9	0,189	0,0603	31,90%	0,002
Zone 10	0,283	0,0692	24,50%	< 1e-04
1995	0,023	0,1200	525,30%	0,849
1996	0,239	0,1140	47,80%	0,036
1997	-0,065	0,1130	175,80%	0,569
1998	0,176	0,1090	61,80%	0,106
1999	0,247	0,1090	44,00%	0,023
2000	0,353	0,1080	30,60%	0,001
2001	0,492	0,1050	21,40%	< 1e-04
2002	0,530	0,1050	19,90%	< 1e-04
2003	0,470	0,1060	22,60%	< 1e-04
2004	0,742	0,1050	14,10%	< 1e-04
2005	0,676	0,1050	15,50%	< 1e-04
2006	0,427	0,1130	26,50%	0,000
2007	0,555	0,1090	19,60%	< 1e-04
2008	0,700	0,1070	15,30%	< 1e-04
2009	0,528	0,1080	20,40%	< 1e-04

Parameters	Estimation	Standard Error	CV(%)	p-value
2010	0,588	0,1110	18,90%	< 1e-04
2011	0,077	0,1130	147,80%	0,498
2012	0,214	0,1080	50,50%	0,048
2013	0,289	0,1100	38,00%	0,008
2014	0,221	0,1150	51,80%	0,053
2015	0,099	0,1100	111,50%	0,370
2016	0,342	0,1120	32,80%	0,002
2017	0,316	0,1130	35,90%	0,005
2018	0,199	0,1130	56,80%	0,079
2019	0,551	0,1080	19,60%	< 1e-04
2020	0,696	0,1090	15,60%	< 1e-04
Feb	-0,126	0,0264	20,80%	< 1e-04
Mar	-0,056	0,0262	47,00%	0,034
Apr	0,048	0,0250	51,70%	0,053
May	0,114	0,0253	22,20%	< 1e-04
Jun	0,230	0,0266	11,60%	< 2e-16
Jul	0,267	0,0267	10,00%	< 2e-16
Ago	0,138	0,0271	19,70%	< 1e-04
Sep	-0,342	0,0371	10,80%	< 2e-16
Oct	-0,385	0,0400	10,40%	< 2e-16
Nov	-0,291	0,0422	14,50%	< 1e-04
Dec	-0,161	0,0298	18,50%	< 1e-04
DFP	-0,090	0,0033	3,70%	< 2e-16

Results of the deviance analysis indicate that all the factors with a higher input degree in the explication given by the factor Year (**Table 3**) played an important role. The other predictors to explain CPUE variations and in order of importance are the factor month, the covariate days out of port (DOP), the factor area (sub-fishing area), and the covariate storage capacity (Storage Cap). The latter has an explanation degree significantly lower when compared to the other predictors (**Table 3**).

Table 3. Deviance analysis table of the fitted compound Poisson-Gamma model

Variables	Degrees of freedom	Deviance	Residual degrees of freedom	Residual Deviance	P-value
Null	---	---	29698	416456	---
Storage Cap.	1	159.5	29697	416297	0,000298
Area	6	1597.2	29691	414700	< 2,2e-16
Year	26	21774.6	29665	392925	< 2,2e-16
Month	11	9332.9	29654	383592	< 2,2e-16
DOP	1	9256.1	29653	374336	< 2,2e-16

Annual estimation of the catch rate indicates a growing trend in the resource abundance and availability up to 2004. In this period the growth has been stable up to 2010 and then it drops to levels similar to those estimated for the beginning of the series and up to 2018. The last two years show an increase in the rate with values around estimations for the period 2004 to 2010 (**Figure 5**).

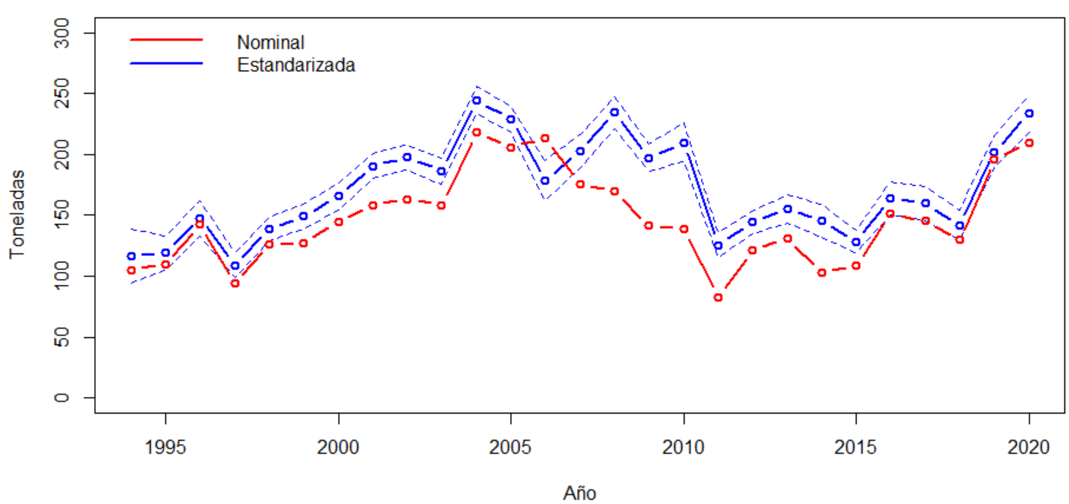

Figure 5. Estimated index of jack mackerel CPUE for the series 1994 to 2020, standardized and nominal. Dotted lines illustrate the confidence interval obtained from the compound Poisson-Gamma model.

Discussion

The model applied allows the explanation of the catch variability fraction of the catch per set for the 1994-2020 period. Although the total of explained Deviance (10.1%) is low due to the high variability between the CPUE index and its covariate, the model make it easier to describe the behavior of the jack mackerel CPUE based on the temporary and spatial factors, as well as on the considered operational covariates.

In comparative terms, the current standardized CPUE series provided by Chile for the SPRFMO jack mackerel stock assessment (**Figure 6**) shows differences that shall be discussed by the SPRFMO Scientific Committee. First, the trends for the period between 1994 and 2006 are discrepant. The current CPUE (blue in **Figure 6**) shows stabilization that is juxtaposed with the increase obtained by the estimated CPUE in this study (orange in **Figure 6**). Secondly, although the reduction of the CPUE since 2007 is observable in both CPUEs, with a promising increase in both series (e.g. from 2011), rates of these reductions and increases are different and less visible in the new CPUE (this study). Finally, absence of catch per set for years after 1994 prevents from reproducing the jack mackerel stock reduction that shows the current series of CPUE between the 1983 and 1996 period.

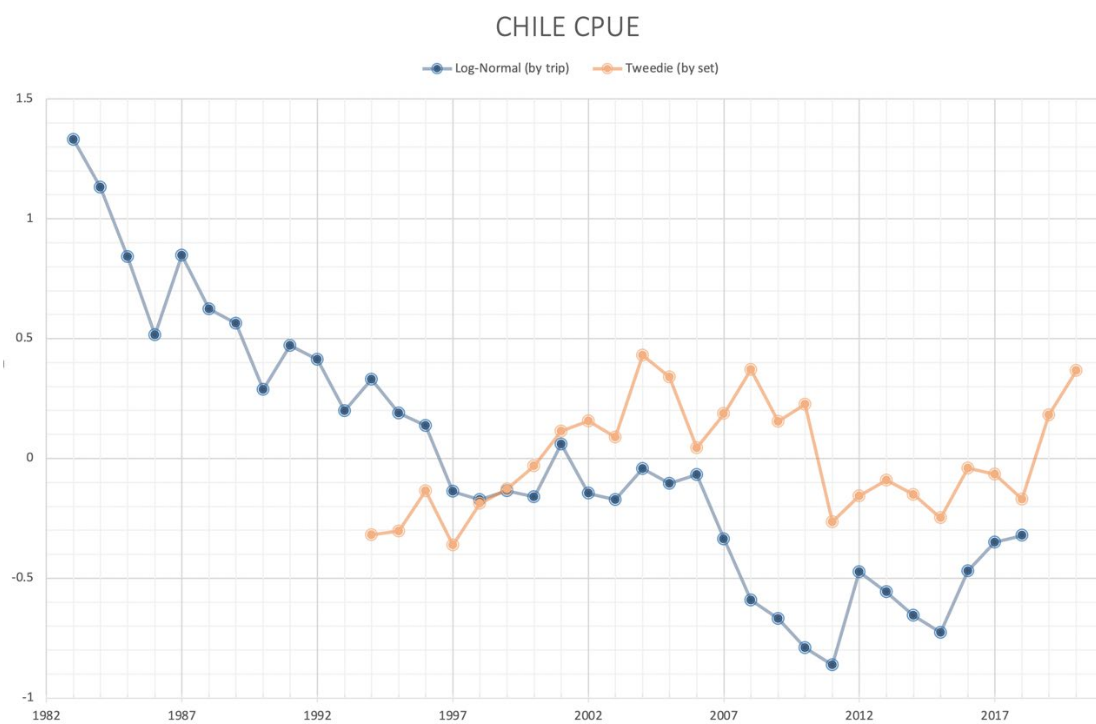

Figure 6. Standardized CPUE by the method Log-Normal (blue) and currently used by the SPRFMO for the jack mackerel stock assessment and the CPUE standardized by a compound model. Please note that the CPUE Log-Normal is based on the catch per

fishing trip while the CPUE obtained in this study is based on catch per fishing set. Both series were scaled to the average value of the CPUE for the 1994-2020 period.

The difference in the estimated trends for both methods may lead to impacts in the jack mackerel stock assessment and to incorrect interpretations on the population variables (e.g. average recruiting) and the amount of management (e.g. MSY). It is distressing not having the information before 1994 that might be included in the standardization of the catch per set since it might prevent the estimation of the stock level before 1982, just before the year the series of CPUE currently used in the stock assessment is used. A way to evade this difficulty is to divide the standardization processes into two series. The first, related to the fishing records based on trips and that allows standardization for the 1982-1993 period; the second, based on the fishing sets that allows a higher resolution for fishing activities in the 1994-2020 period, as shown in this paper. In terms of its used in the assessment model, this shall make it easier to adopt decisions on the catchability for each time series and on the comparability amongst assessment periods.

Chile will continue with standardization exercises aimed at achieving a conjunction between the fishing records available before 1994 (catch per fishing trip) and those used in this study (catch per fishing set). However, the method formed by standardization detailed in this document still requires improvements. For example, the model indicates a poor log-linear relationship between the covariate holding capacity with the CPUE (see Table 3). This could be explained by the use of generalized additive models (GAM) as an alternative.

Another point has relation with the dependence degree of the set on the fishing trip (or vessel). This situation was not taken into account in this study. Under this framework and for the next studies, Chile will explore the use of alternative models that would allow the incorporation of mixed effects so the additional variation degree is collected due to the trip-vessel cluster.

Lastly, it is important to mention that the results of this study shall be looked at very carefully since they standardize a sampling and an effort unit different to the one used in the previous work.

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ANEXX

Introduction

The Tweedie class of distribution are exponential dispersion models closed under scale transformations (Jϕrgensen, 1997), known for its role in the generalized lineal models, the representation of the exponential disperdsion model of the parameters is associated for,

$$f(\mathrm{y}|\phi,\theta) = a(y,\phi)\exp\left\{\frac{y\theta-\kappa(\theta)}{\phi}\right\} \qquad (1)$$

Where a and κ are known functions, θ is the natural parameter and $\phi > 0$ is the dispersion parameter. For this family of distributions, it is known that $\mathbb{E}[\mathrm{Y}] = \mu = \kappa'(\theta)$ and $\mathbb{V}(\mathrm{Y}) = \phi\,\kappa''(\theta)$, where κ' and κ'' correspond to the first and second derivate of the function κ with respect to θ. The variance function, identified with $\mathbb{V}(\mu)$, may be represented as a function of μ and corresponds to $\kappa''(\theta)$. Hereafter, we will focus on the exponential dispersion model with a variation of the power, given by,

$$\mathbb{V}(\mathrm{Y}) = \phi\mu^{\boldsymbol{p}} \qquad (2)$$

Depending on the value of the parameter $p \in\,]-\infty\,;0\,] \cup [1\,;\,\infty[$, index of the Tweedie distribution, underly several other known probability distributions such as the Normal, Poisson, compound Poisson, Gamma, gaussian inverse and stable distribution are shown in **Table 1**. Therefore, if a random variable Y follows a Tweedie distribution law, it is then designed by $Y{\sim}Tw_p(\mu,\phi\,)$.

The study focuses on the exponential dispersion model with a variance function given by (2), where the support is for the parameter $p \in\,]1\,;2[$. This particular distribution is generated by a compound Poisson-Gamma composed that has a probability mass in the origin accompanied by a continuous distribution biased in the positive real line, which is also known as a Poisson distribution composed of Tweedie or simply the compound Poisson distribution. Extensive applications of this distribution have been found, mainly regarding the context of generalized lineal models (GLM), on a wide variety of fields where continued data with exact ceros emerge.

Table 1: Summary of the Tweedie exponential dispersion models

Distributions	Value of parameter $\boldsymbol{p}$
Estable extremo	$p < 0$
Normal	$p = 0$
No existe	$0 < p < 1$
Poisson	$p = 1$
Poisson Component	$1 < p < 2$
Gamma	$p = 2$
Estable positivo	$2 < p < 3$
Gaussiana Inverse	$p = 3$
Estable positivo	$p > 3$
Estable extremo	$p \to \infty$

Definition

$N, X_1, X_2, \ldots, X_N$ is an independent random variable sequence that represent the number of sets with fishing and catches obtained, respectively, where,

$$Y = \sum_{i=1}^{N} X_i \quad ; \quad N \sim Poisson(\lambda) \quad ; \quad X_i \sim Gamma(\alpha\,,\gamma) \quad ; \quad N \perp X_i \tag{3}$$

Where $\mathbb{E}[\mathrm{N}] = \mathbb{V}(\mathrm{N}) = \lambda$; $\mathbb{E}[X_i] = \alpha\gamma$; $\mathbb{V}(\mathrm{N}) = \alpha\gamma^2$.

Please note that in definition (3), if $\mathrm{N} = 0 \Longrightarrow \mathrm{Y} = 0$, which allows that the distribution has a probability mass in zero. When $\mathrm{N} > 0$, the variable answer Y is an amount of random variables i.i.d. gamma, which means that $Y|N \sim Gamma(N\alpha\,,\gamma)$, where the joint distribution of the Poisson random variables and Poisson compound, is given by,

$$f(\,\boldsymbol{y}\,|\lambda, \alpha\,,\gamma) = f(Y, N\,|\lambda, \alpha\,,\gamma) = f(Y|N, \alpha\,,\gamma) f(N|\lambda)$$

$$f(\,\boldsymbol{y}\,|\lambda, \alpha\,,\gamma) = \begin{cases} \exp(-\lambda)\,, & \boldsymbol{y} \in (0{,}0) \\ \dfrac{\mathrm{y}^{N\alpha-1}\exp(-\,y/\gamma)}{\Gamma(N\alpha)\gamma^{N\alpha}} \cdot \dfrac{\lambda^N \exp(-\lambda)}{N!}\,, & \boldsymbol{y} \in (\mathbb{R}^+ \times \mathbb{Z}^+) \end{cases} \tag{4}$$

Thus, composed Poisson-Gamma distribution has a probability mass in zero, followed by continuous distribution biased towards the right. This specific feature makes it ideal for modeling continuous data with exact zeros that usually arise in many fields applied to science.